524 Summer syllabus

Ahead of the first meeting everyone should read III.1 on their own.

(suggested problems: III 2.1a, 2.2, 2.3, 2.4, 2.7)

- quick review of important concepts from section 1
  (e.g. "enough injectives", derived functors)

- Usefulness of cohomology in AG: Sheaf cohomology as right derived functors of global section functor.

- Example of usefulness: $x \in X$, $\mathcal{L}$ invertible.

$$0 \to \mathfrak{m}_x \to \mathcal{O}_X \to i_* \mathcal{O}_x \longrightarrow 0$$

$\uparrow$ ideal sheaf of x

$\longleftarrow$ skyscraper sheaf

$\uparrow$ structure sheaf of point

$$0 \to \mathfrak{m}_x \otimes \mathcal{L} \to \mathcal{L} \to i_* \mathcal{O}_x \otimes \mathcal{L} \longrightarrow 0 \quad \text{is still exact}$$

$\mathcal{L}$ is globally generated at x $\iff$ this map is surjective after taking global sections. (Think through this: $i_* \mathcal{O}_x \otimes \mathcal{L} \cong \mathcal{L}/\mathfrak{m}_x \mathcal{L} \cong \mathcal{O}_x/\mathfrak{m}_x$)

Thus, global generation depends on $H^0(i_* \mathcal{O}_x \otimes \mathcal{L}) \to H^1(\mathfrak{m}_x \otimes \mathcal{L})$ being the zero map

- III 2 : skip most proofs, sketch pf of 2.7.

Mon 5/9 (suggested problems: 3.1, 3.2, 3.3)

- III 3: Focus on Thm 3.7. Can skip/sketch other proofs as necessary.

Wed 5/11 (suggested problems: none—just read section carefully on your own)

- II 8: Focus on sections "sheaves of differentials" and "nonsingular varieties" — just to get some of the basics. If you want, you can use another source for this material. (Bertini's Thm is important!)

Fri 5/13 (suggested problems: 4.1, 4.3, 4.7)

- III 4: heavy emphasis on examples — e.g. work out 4.0.3 in great detail. Leave out proofs where necessary in order to devote time to examples/computations.

Mon 5/16 (Suggested problems: III 5.1, 5.2, 5.5)

- III 5: just state Thm 5.1 (go through proof on your own), prove 5.2, prove 5.3 (time permitting)

* time permitting!

Wed 5/18 (problems: III 6.1 (hard, but important), 7.1)

- III 6: Definitions and basic properties — don't prove anything here.

- III 7: Up through Cor 7.8: skip most proofs probably. Defs + statements of Thms are most important here —

specifically: def of dualizing sheaf, Prop 7.5, Cor 7.7.
7.15 (Kodaira vanishing) is worth mentioning too.

Continued reading: II 7 ("Proj, $\mathbb{P}(\mathcal{E})$, and Blowing up"
subsection), chapters IV and V (These will seem fun
and super concrete after chapters II + III.)